Maximal Poisson-disk Sampling via Sampling Radius Optimization

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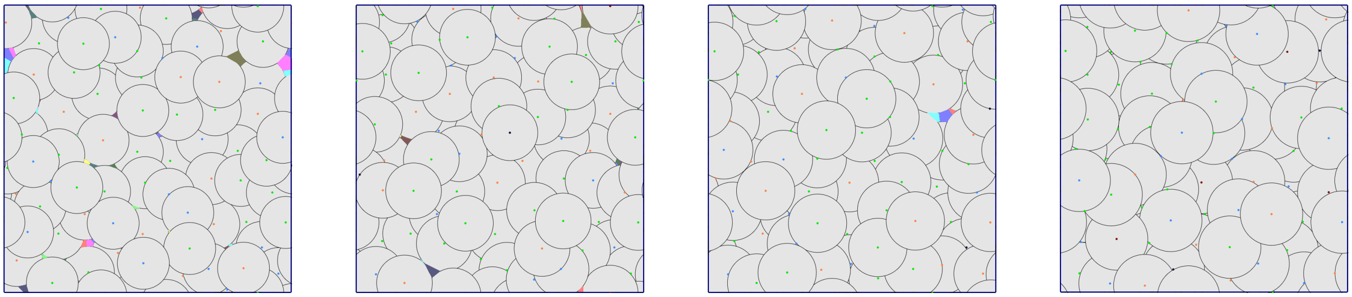

Figure 1: *The work flow of our method. From left to right: initial point set, the result after 50 iterations, the result after 100 iterations, and the final result. By iteratively adjusting the position of points and the sampling radius, the gap regions (colorized polygons) become smaller and smaller. Finally, the sampling becomes maximal.*

Abstract

Maximal Poisson-disk Sampling (MPS) is a fundamental research topic in computer graphics. An ideal MPS pattern should satisfy three properties: *bias-free, minimal distance, maximal coverage*. The classic approach for generating MPS is dart throwing, but this method is unable to precisely control the number of samples when achieving maximality [Ebeida et al. 2011]. Sample elimination [Yuksel 2015] is an recently proposed algorithm that could generate Poisson-disk sets with an exactly desired size, but it cannot guarantee the maximal coverage. In this work, we propose a simple 2D MPS algorithm that can precisely control the number of samples, while meeting all three criteria simultaneously. Unlike previous conflict-based methods, our algorithm controls the number of samples by dynamically adjusting sampling radius.

Keywords: Maximal Poisson-disk sampling, Delaunay triangulation

Concepts: •**Computing methodologies** → **Computer graphics;** *Geometric processing;*

1 Our Approach

Core idea. Given the desired sample number N and the sampling domain Ω, we first generate an initial point set $\mathbf{X} = \{\mathbf{x}_i\}_{i=1}^N$ using an estimated sampling radius, which is a little smaller than theoretical value of using N samples. Then we construct a Delaunay triangulation $DT(\mathbf{X})$ and iteratively optimize the point distribution. In each iteration, the shortest edge E is selected in $DT(\mathbf{X})$ and one endpoint of E is removed from $\mathbf{X}$. We then dynamically update $DT(\mathbf{X})$ and carry out a gap detection operation [Yan and Wonka 2013]. Subsequently, we extract gap regions using the gap processing algorithm [Yan and Wonka 2013] and insert a new point into $\mathbf{X}$ by performing dart throwing in a random gap primitive. This algorithm converges when there are no gap triangles in $DT(\mathbf{X})$ any more after removing one endpoint. In this case, the new point will be positioned at the current largest circumscribe center.

Endpoint removal. To determine which endpoint of the shortest edge should be removed, we select the end point with the largest neighborhood-averaged area. The neighborhood-average area is defined as $A_{avg} = \frac{1}{T(\mathbf{x}_i)} \sum_{i=1}^{T(\mathbf{x}_i)} A_i$, where $T(\mathbf{x}_i)$ is the number of incident triangles, and A_i is the area of each incident triangle.

Arbitrary boundary conditions. To handle arbitrary input boundaries, we first generate samples at the sharp corners and make these points fixed during subsequent optimization process. Next, we integrate a feature-preserving operation into above framework: after each iteration, samples whose Voronoi cells intersect with the boundary will be projected onto the boundary.

2 Results and Future work

We compare our results with the recent representative Poisson-disk sampling algorithms in Fig. 2, including *Sampling Elimination* (SE) [Yuksel 2015], *Farthest Point Optimization* (FPO) [Schlömer et al. 2011] and *Maximal Poisson-disk Sampling* (MPS) [Ebeida et al. 2011; Yan and Wonka 2013]. The spectral analysis are generated by PSA [Schlömer and Deussen 2011] software. We see that our results have similar spectral properties to MPS.

We also compute the quality of sampling and the corresponding triangulation in periodic domain as shown in Tab. 1. The reader is referred to [Yan et al. 2015] for the meaning of these measures. It shows that FPO has the best geometric properties, but the FPO algorithm is deterministic and it is the slowest; while SE is fastest among all the competitors, it has the lowest qualities. Both the spectral and geometric properties of our results are quite similar to MPS, while we can explicitly control the number of samples.

Fig. 3 visualizes the comparison with SE [Yuksel 2015], which indicates that SE is not maximal. We also compare the convergence speed of our method with FPO, as show in Fig. 4. A full iteration means that all the samples are moved once. We find that the sampling radius of our method becomes stable after several iterations(< 10), which means the maximality is achieved. Fig. 5 shows two results of sampling in polygonal domains.

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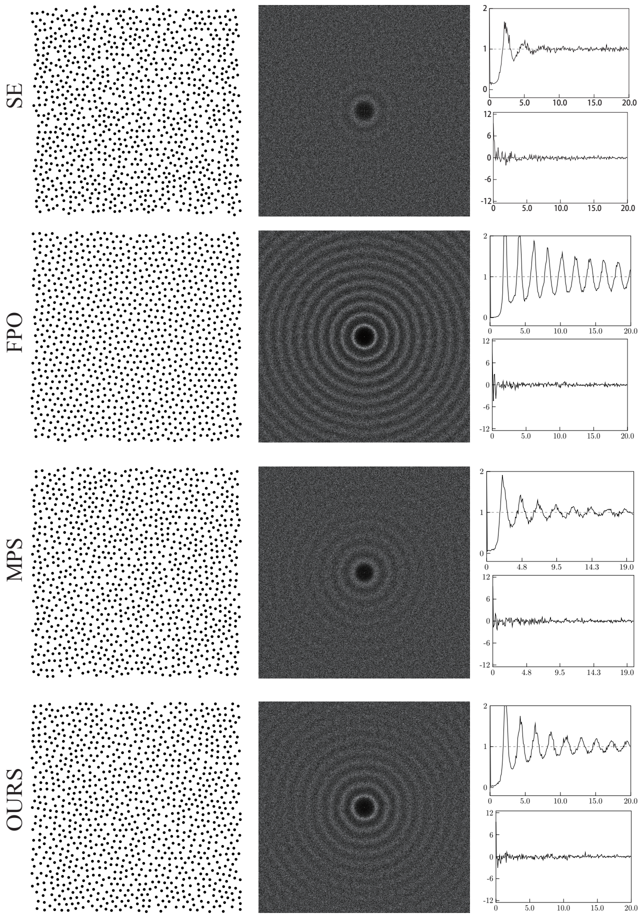

Figure 2: *Comparison of the spectral properties.*

Table 1: *Statistics of the sampling and the triangulation quality in periodic domain.*

method	δ_X	Q_{min}	Q_{avg}	θ_{min}	$\overline{\theta}_{min}$	θ_{max}	$\theta_{<30^\circ}$	$\theta_{>90^\circ}$	$V_{567}\%$
SE	0.652	0.412	0.778	20.59	42.92	126.42	4.83	21.72	94.14
FPO	0.925	0.567	0.856	35.12	50.90	107.51	0.00	6.50	99.61
MPS	0.780	0.487	0.806	30.19	45.30	117.11	0.00	15.07	96.53
OURS	0.855	0.514	0.833	30.21	47.97	113.85	0.00	12.60	98.30

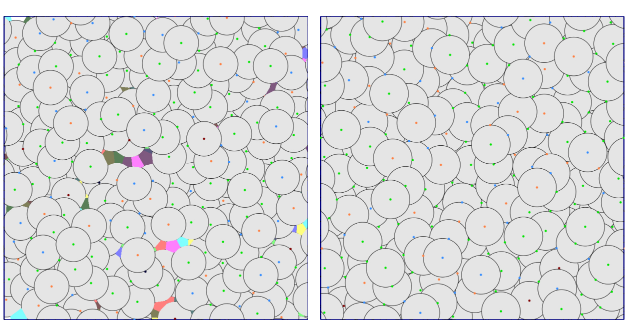

Figure 3: *Comparison of sampling results using $N = 200$ points. Left: the result of sample elimination [Yuksel 2015], where colorized polygons indicate gaps; right: our method without gaps.*

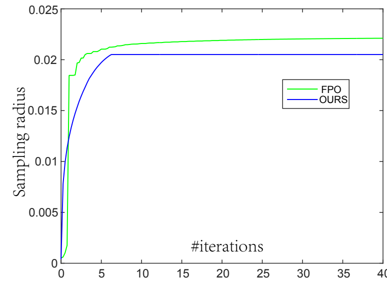

Figure 4: *Comparison of the convergence speed of our method and FPO using the same randomly generated initial point set (with N=2k).*

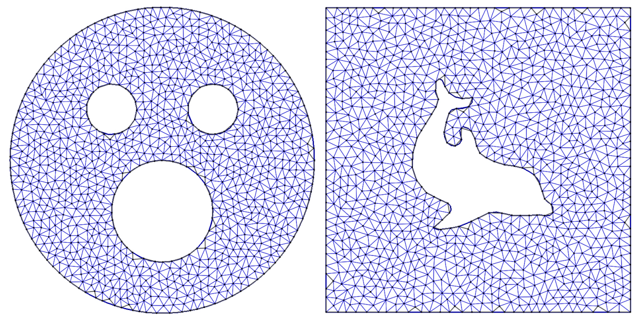

Figure 5: *Sampling results of several arbitrary boundaries: Face and Dolphin.*

In the future, we plan to extend this approach to adaptive sampling as well as surface sampling. We would also like to implement a GPU version of the proposed algorithm.

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